Bio-inspired Group Modeling and Analysis for Intruder Detection in Mobile Sensor/Robotic Networks

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***Abstract*—Although previous bio-inspired models have concentrated on invertebrates (such as ants), mammals such as primates with higher cognitive function are valuable for modeling the increasingly complex problems in engineering. Understanding primates' social and communication systems, and applying what is learned from them to engineering domains is likely to inspire solutions to a number of problems. This paper presents a novel bio-inspired approach to determine group size by researching and simulating primate society. Group size does matter for both primate society and digital entities. It is difficult to determine how to group mobile sensors/robots that patrol in a large area when many factors are considered such as patrol efficiency, wireless interference, coverage, inter/intragroup communications, etc. This paper presents a simulation-based theoretical study on patrolling strategies for robot groups with the comparison of large and small groups through simulations and theoretical results.**

***Index Terms*—Bio-inspired communication, robot grouping, wireless networks.**

I. Introduction

MOBILE robots equipped with sensors are able to cooperatively work together via wireless communication technologies in order to achieve and obtain surveillance teaming as well as task accomplishments in a large, complex field [1], [2]. The major challenges of communication in the large and complex field are considered that the number of mobile sensors is insufficient for a constantly available network used by intra/intergroups. While each group may be able to maintain communication within the group at all times, a complete path for constant end-to-end data communication for any pairs of source and destination in different groups may not exist. There are always unmonitored locations due to the limited number of mobile sensors/robots that cannot monitor and cover the whole field. In order to solve such a problem, the mobile robots/sensors need to patrol the entire field in order to cover it completely. Unfortunately, we are uncertain as to how to group the robots/sensors to achieve a low cost. The size of the robot/sensor groups could be either large or small. It is not easy to intuitively determine which grouping size is the more efficient.

A similar choice exists in primate society as well. Rhesus macaques and titi monkeys are two kinds of primates that usually live in groups in order to supervise their territory, defend against intruders, and search for food. Rhesus macaques live in large groups that normally contain 10–80 individuals, regardless of habitat type [3]. The members in the group communicate via facial expressions, body postures, and vocal communication [3], [4]. Communication within each group is complicated because of the large number of members in the group. Titi monkeys, however, live in small groups that only consist of the parents and their offspring [5], [6], [32]. Each group of titi monkeys contains a total of 2–7 animals [6].

Besides rhesus macaques and titi monkeys, there are many other primate types that have solved the given patrol problem. However, rhesus macaques and titi monkeys solve the territory-patrolling problem in two fundamentally different ways: large group and small group, respectively. Inspired by these interesting strategies in nonhuman organisms in nature, we can apply similar strategies and techniques to artificial systems, such as the robot groups patrolling in certain environments. Throughout this paper, we utilize studies of grouping in primate species to study teaming and surveillance, provide a study on the performance of a large and small group, and generate video demos to show the process of the simulation. Similar to primate society, given a certain number of robots, we can: 1) group them into large groups that contain a relatively large number of robots as shown in Fig. 1(a); or 2) group them into small groups that contain a relatively small number of robots as shown in Fig. 1(b). The main purpose of this paper is to study and simulate the behaviors of the primate groups and observe the tradeoffs of group size. Our goal is to study the benefits and tradeoff between the large and small groupings.

The determination of group size in a robotic network has been examined in some related works, and it shows that group size affects the efficiency as well as the system cost in a robotic network [7]. Rekleitis *et al*. [8] claim that the group size affects the accuracy of the localization for different classes of robots.

Manuscript received February 25, 2013; revised December 27, 2013; accepted April 16, 2014. Date of publication May 16, 2014; date of current version December 15, 2014. This work was supported in part by the National Science Foundation under Grant CCF-0829827, in part by the Macau Science and Technology development fund under Grant 008/2010/A1, and in part by the University of Macau Multiyear Research Grants. This paper was recommended by Associate Editor Z. Wang.

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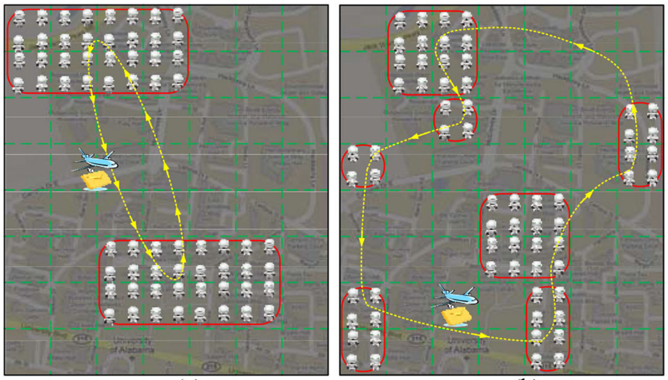
Digital Object Identifier 10.1109/TCYB.2014.2320717



(a) (b)

Fig. 1. 2-D area for robots/sensors' moving and patrolling. (a) Large group. (b) Small group.

Lerman and Galstyan [9] examine the importance of group size as well, and find that the performance could be improved as the group size grows, but the interference caused by the large number of robots decreases the performance of each robot individually. The authors of [10]–[12] research the group size as well to show the effect of group size on the network's performance. Similar to this paper, Krieger *et al*. [11] mention a bio-inspired approach in a robotic network to show the relationship between group size and efficiency. These works convince us that the research on the relationship between group size and system performance of a mobile sensor/robotic network is valuable. Moreover, there are works about intruder detection and path determination in sensor/robotic networks being researched. An observation selection problem was considered in [13] in order to optimize some objective function including intruder detection for robotic groups through a submodular saturation algorithm. A near-optimally solution informative path determination was proposed in [14] while considering the group size of the robots.

For another part of this project of our paper (supported by the same U.S. NSF Grant CCF-0829827), communication protocols and behavior algorithms, such as bio-inspired message-based communications and bio-inspired behavior-based communications were proposed and experimented in our works [15], [16]. These works show how factors specific to robot groups such as interference and message handling using various environmental configurations and communication paradigms [16]. Robot movements are determined by the behavior observation of other robots, message communication, and the environment [15]. For another part of this project of our paper, an important communication method called scent-marking, which is used by primates to demonstrate their purpose, to their social rank, and to show their aggression, was proposed and researched in sensor/robotic networks [17]. Delayed-and-relayed communication that is inspired by primates was also proposed, and the communication method achieves leaving a message for a robotic node or to leave a robotic node to be picked up by other robots at a later time [17]. We as a team for the NSF grant did some of the aforementioned work and we have explained communication protocols and behavior algorithms in the papers. However, this paper only focuses on the group size in robotic networks.

For another part of this project of our paper, we studied the coalition formation of robots for detecting intrusions using game theory in a group of three robots that detect and capture intrusions in a closed curve loop [33].

This paper specifically considers how the two types of primates organize themselves (small versus large groups), analyzes how these strategies can be applied under our assumptions, and presents an analysis and a computational geometry simulation to show the benefit and trade-off of both large and small groups. The observation and assumption about the primate society is that some species live in large groups, while some other species live in small groups. Unlike flocks or shoals, the group size of the primate society is usually not extremely large. There are normally tens of members in a primate group. Moreover, although the primate group size changes due to birth and death, the group usually does not split to smaller groups or merge with other groups to form a larger group. For social purpose, a primate group may form as a one-male or multimale/multifemale group which is determined by the characteristic of the species. In other words, a specific primate group barely dynamically determines its group size based on some trade-offs or environments.

There is a strenuous need when having to handle a large and complex area with a limited number of mobile sensors, which cannot monitor or cover the whole area, so there are holes (unmonitored places) all the time. However, movements of mobile sensors reduce holes in a temporal sense. In practice, there might not be central control, and therefore sensors should cooperatively work together via local wireless communications. Static sensors or real-time deployed sensors will also help to achieve the surveillance goals.

When large groups of mobile sensors/robots are used [Fig. 1(a)] within any group, mobile sensors/robots can form connected or disconnected *ad hoc* networks. Due to the fact that the surveillance area is large and complex, mobile sensors/robots within the same large group are often disconnected in terms of communication. In another case, as illustrated in Fig. 1(b), when small groups of mobile sensors are used, mobile sensors can form a full-connected one-hop wireless network individually within each group. The large, complex area is divided into smaller regions that can be further divided into even smaller areas, like smaller cells in cellular networks.

Therefore, bio-inspired modeling helps us determine which aspects of primate social and group behavior we should translate and use to best address the problems discussed in this paper. Regardless of how we group the robots/sensors, it is imperative that the following challenges are solved in order to evaluate the performance of the grouping: patrol time [18], communication interference inside a group [19], intruder defense [20], and communication among all the groups [21]. The following paragraphs explain these challenges in this paper.

The area in which the robots/sensors are moving and patrolling is abstracted as a 2-D rectangle, as shown in Fig. 1. The entire area over which the robots/sensors are patrolling is called field. The robots/sensors are supposed to supervise all parts of the field. In this case, no matter how the robots/sensors are grouped, the coverage of the field by the robots/sensors should be 100%. It is time consuming for the monkeys to supervise all their territory. Similarly, it is time consuming for the robots/sensors to cover the entire field. The length of this time, called patrol time, influences how quickly the robots can react to the environment of the field changing, i.e., the invading of the intruders.

Communication between two monkeys is diminished by obstacles and by vocal communication between other monkeys. This intragroup interference in primate society is also applied to the robot groups in which wireless interference takes place. The robots/sensors in the field are essentially wireless sensor networks, so we have to address the problem of lowering the network performance, i.e., interference [22]. Interrupting the communication among the robots/sensors in a group is an interference that damages the consistency of the robots/sensors when heading for a destination and then reduces the robots' velocity and ability to react to intruders.

One purpose of the robots/sensors supervision of the field is to detect and remove (or at least deport) intruders, as in the monkey group. The size of the robot group affects the cooperation and efficiency of the robots/sensors. Intuitively, we know that interference in large groups is more severe than in small groups and that it is easier for a large group to clear an intruder than a small group. However, we do not know how to determine group size in order to balance interference and intruder defending.

Intergroup communication is an essential issue within this paper for the following reasons: 1) each group of robots/sensors should be aware of the coverage of other groups in order to determine its patrol area and 2) we need to gather information from all the groups, such as the coverage of a group, the number of intruders a group encounters, etc. Therefore, a courier, e.g., an AV or mobile robot, is introduced in this paper (shown in Fig. 1). The courier flies among all the groups constantly in order to achieve intergroup communication.

In this paper, we investigate several approaches to realize the architectures and how these bio-inspired communication and networking approaches support the surveillance teaming and task accomplishments. We also study the impact of surveillance teaming and control on communication metrics. We create a model based on primate groups of different sizes and cohesiveness. In either case, the group can reorganize in response to messages or events. We build a model of searching and communication that best fits primate behavior in different-sized groups. Agent-based model (ABM) is a logical first step in modeling animal social and group behavior since it focuses on modeling individuals and their interactions with simple rules [23]. Nevertheless, the design of ABM is not limited to the implementation of simple rules as is the case in insect models of group and swarm behavior. Indeed, the group has the capability to implement agents capable of learning as well as individual recognition and memory and those that have personality traits that affect their behavior. In this paper, ABM is conducted on an open source software tool called MASON [24].

The contributions of this paper are stated as follows.

1) We use bio-inspired modeling and simulation to help us determine which aspects of primate group behavior best addresses the problems discussed in this paper.
2) We study large and small grouping concerning communication.
3) A standard to evaluate the performance of large and small grouping is discovered by examining problems in both robot computing and wireless networks.
4) Given a certain number of robots/sensors, this paper compares the two ways of grouping (small grouping and large grouping) by: a) patrol time of the entire field; b) communication interference in a group; c) intruder defense ability; and d) iteration time of the courier.
5) Our modeling and simulation can be extended to other types of sensor networks.
6) We provide theoretical studies that model the large and small grouping in Section IV. Significant insights can be obtained from these theoretical studies.
7) We provide video demos that show the robots moving and beating the attackers.

The rest of this paper is organized as follows. There are mainly two parts in our paper: the simulation and the theoretical study. For the simulation, Section II provides the description and definition of the problem, while Section III provides the simulation results that give the comparison of the performance of large and small grouping. For the theoretical study, Section IV provides a theory deduction to model large and small grouping based on assumptions that are different from Sections II and III. Related works are presented in Section V. Finally, we conclude this paper in Section VI.

II. Group Behavior And Problem Definition

In this section, we describe the behavior of the mobile sensors/robots, present a model that describes the properties and behaviors of mobile sensors/robots, and define the problems that need to be solved. We give a description of our problem, however, it is lacking a solution. The research is conducted by simulations and theoretical deduction, presented in Sections III and IV, respectively.

A. Problem Description

For convenience, the rectangle field is divided into several smaller rectangle areas, called sub fields (see the small dash-line rectangles in Fig. 1). A group is a set of mobile sensors/robots that can communicate among each other by one hop as well as multiple hops of wireless communication, which are depicted as red circles in Fig. 1. A sub field has the possibility of being monitored by one or more groups. Mobile sensors/robots in a group need to "walk" through certain sub fields to monitor and detect intruders in these sub fields. These robots are equipped with sensors to detect intruders and with certain facilities to terminate their attacks. Assume that the robots/sensors in different groups are not able to communicate with each other directly since the distances might be longer than the communication range. A courier [an unmanned aerial vehicle (UAV) or a mobile robot] is used for communication among these groups along a mutative route that connects all the groups (see the yellow dash-line circles with arrows in Fig. 1). The courier receives data from the group that it is flying over and sends data received from other groups to this group.

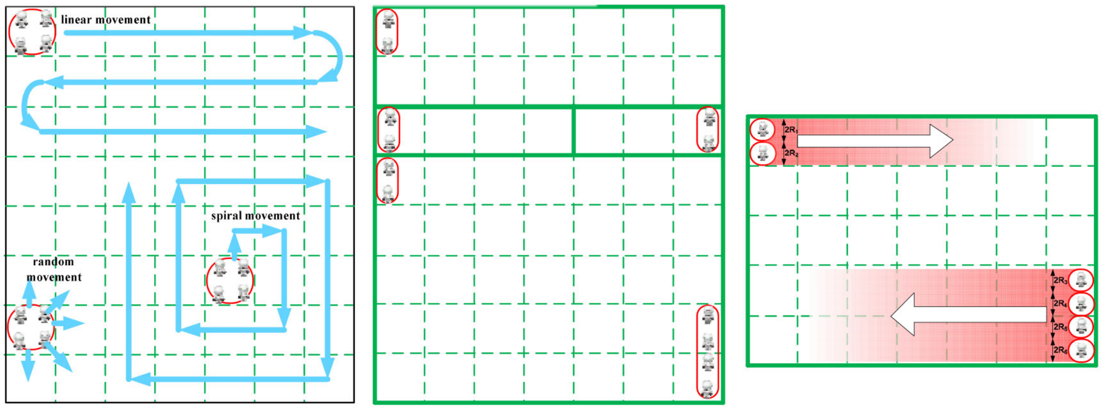


Fig. 2. Movement. (a) Group movements. (b) Sub fields. (c) Movement in a sub field.

There are many manners for a group to move, including linear movement, random movement, spiral movement, etc., as shown in Fig. 2(a). However, we assume the linear movement in this paper. The dash-line rectangles in Fig. 2(a) are only used to illustrate the movement trace of the groups.

Intruders may invade the field. Assume that the probability of one robot to find and beat the intruder is a constant, and therefore the large group more easily finds and beats the intruder than the small group does since there are more robots/sensors in a large group.

Naturally, we know that a large group is good at finding and beating the intruder, but the large number of robots/sensors in the group results in a longer delay, more collisions, larger communication overhead, and failure of data transmissions. The delay, collisions, communication overhead, and failure of data transmissions in a small group are not as severe as in a large group, but the robots/sensors in the small group cannot collaborate as powerfully as the large group due to the limited ability of each robot. Our goal is to study the benefits and tradeoff between the large and small groupings.

B. Model

This subsection provides a model to describe the moving field of the robots/sensors, to regulate the movement of the robots/sensors, and to calculate the performance of each moving method. The proposed movement model was inspired by the work in [1], but they are significantly different.

The field of the robots/sensors is denoted by a 2-D space on the ground and is divided into m sub fields, as shown in Fig. 2(b). In Fig. 2(b), there are four sub fields which are encircled by bold lines. The dash-line rectangles in Fig. 2(b) are only used to illustrate the movement trace of the groups and help show the area of the subfields. A group of mobile robots/sensors moves only within a sub field, but it is possible for more than one group of robots/sensors to move within a sub field. A courier commutes between groups for intragroup communication.

Let w_k and h_k denote the width and height of the kth sub field whose area is $w_k \times h_k$. We assume that the robots/sensors in a group move together and that they move linearly except upon reaching the border of the sub field, as shown in Fig. 2(a). Fig. 2(c) shows a sub field (the largest one at the bottom) of the field in Fig. 2(b) and shows how robots/sensors move in a sub field. There are two groups in Fig. 2(c), and the robots/sensors in each group stand in a line and move together. The region that a group of robots/sensors covers is a rectangle. When this group reaches the edge of its sub field, these robots/sensors move vertically to a new horizontal line and then horizontally in the opposite direction [see linear movement in Fig. 2(a)].

We use discrete time in our model, denoted as $t = \{1,2, \ldots, T\}$. Assume that in the kth sub field there are g_k groups of robots/sensors, and let $v_{k,i,t}$ denote the moving velocity of all the robots/sensors in the ith group at time t, where $i = 1,2,\ldots, g_k$. Let $s_{k,i}$ denote the total number of robots/sensors from the 1st group up to the ith group in the kth sub field. Assume that the detection/sensing range of each robot in the ith group of the kth sub field is a disk with the radius $R_{k,i}$ centered at the robot.

A successful communication is achieved by a pair of robots successfully sending and receiving a data package through a one-hop connection. Such a communication may be failed due to traffic, collisions, and other factors. Therefore, the probability of a communication being successful is affected by the network conditions. For simplicity, we assume that the probability of a successful communication of each robot is the same in a group, and we denote the successful communication probability as p. It is determined by the size of the group and the data traffic in the group. The moving velocity $v_{k,i,t}$ is determined by the delay time of the communication, the turning time of a robot, the probability of successful communication, etc. The moving velocity should be constant if there is no delay in communication and the probability of successful communication is 1.

If these robots/sensors move as shown in Fig. 2(c), the region that a group of robots/sensors covers is a rectangle. In the ith group in the kth sub field, in a time period T, the distance that one single robot/sensor moves is $\int_0^T v_{k,i,t}\, dt$. Hence, in a time period T, the area that the 1st group in the kth sub field covers is $\left(2\sum_{i=1}^{s_{k,1}} R_{k,i}\right) \times \int_0^T v_{k,1,t}\,dt$. In a time period T, the area that all groups cover is $\theta = \left(2\sum_{i=1}^{s_{k,1}} R_{k,i}\right)\int_0^T v_{k,1,t}\,dt + \left(2\sum_{i=s_{k,1}+1}^{s_{k,2}} R_{k,i}\right)\int_0^T v_{k,2,t}\,dt + \ldots + \left(2\sum_{i=s_{k,g_{k-1}}+1}^{s_{k,g_k}} R_{k,i}\right)\int_0^T v_{k,g_k,t}\,dt$.

For convenience, if the average velocities of all the robots/sensors in the kth sub field is v_k, then $\theta = 2\sum_{i=1}^{s_{k,g_k}} R_{k,i} \times v_k \times T$. Therefore, the time that all of the groups of robots/sensors in the kth sub field require to cover their entire sub field is $\mathrm{Time}_k = (w_k \times h_k) \Big/ \left(2\sum_{i=1}^{s_{k,g_k}} R_{k,i} \times v_k\right)$, where v_k is the average velocity of all of the robots/sensors in the kth sub field.

The movement of groups of robots/sensors in each sub field is simultaneous over the entire field. Hence the time that all the groups of robots/sensors require to cover the entire field is the maximum time among all of the sub fields. $\mathrm{Time} = \max_{k=1,2\ldots,m} \mathrm{Time}_k = \max_{k=1,2\ldots,m} \left[(w_k \times h_k) \Big/ \left(2\sum_{i=1}^{s_{k,g_k}} R_{k,i} \times v_k\right)\right]$, where v_k is the average velocity of all of the robots/sensors in the kth sub field.

If we assume that any intruder in a sub field can be found by mobile robots/sensors, we have the following results. If there is only one intruder in each sub field, for each group among all of the g_k groups in the kth sub field, the probability of the ith group encountering this intruder is $1/g_k$ on average. If we assume the probability of a robot to beat an intruder to be q, then the probability of the 1st group of the kth sub field finding and beating an intruder is $q_{fb,k,1} = 1 - (1-q)^{S_{k,1}}$. Similarly, the probability of the ith group ($i > 1$) of the kth sub field finding and beating an intruder is $q_{fb,k,i} = 1 - (1-q)^{S_{k,i} - S_{k,i-1}}$, where $i = 1, \ldots, g_k$. The probability of the ith group encountering this intruder is $1/g_k$, and thus the probability of the kth sub field finding and beating an intruder is $q_{fb,k} = \sum_{i=1}^{g_k} q_{fb,k,i} \Big/ g_k$.

Hence, the probability of the entire field finding and beating an intruder is

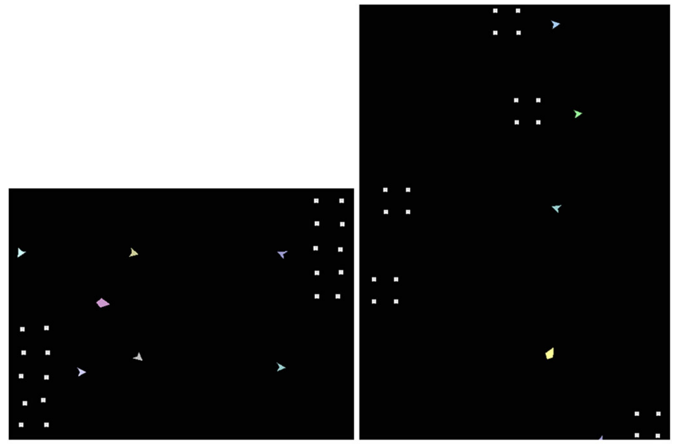
$$q_{fb} = \prod_{k=1}^{m} q_{fb,k} = \prod_{k=1}^{m}\left(\frac{1}{g_k}\sum_{i=1}^{g_k} q_{fb,k,i}\right) = \prod_{k=1}^{m}\left(\frac{1}{g_k}\sum_{i=1}^{g_k}\left(1-(1-q)^{S_{k,i}-S_{k,i-1}}\right)\right).$$

III. Simulation Studies

We simulated robots/sensors grouping and moving, generated simulation videos and screenshots, and analyzed simulation results. The assumptions in the simulations are the same as those in Section II unless stated otherwise. Both the simulation study in Section III and the theoretical study in Section IV share the same problem definition, background, and motivation. We expect the simulation and theoretical study to give us understanding about grouping from different aspects.

A. Simulation Setup

In order to generate graphic simulation results, we wrote our simulation programs based on MASON [24], a fast discrete-event multiagent simulation library core in Java.



(a) (b)

Fig. 3. Screenshots of video clips for robot movement. (a) Two large groups. (b) Five small groups.

In our simulations, the field is a square with an area of 150 × 150 units. Our work focuses on group size determination of the robotic group, and thus we do not consider the dynamics as a main part of the simulation. Therefore, we abstract the robots in the simulation as an object that moves in the patrol field. Unlike flocks or shoals, a primate group usually consists of at most tens of members. Our work is based on the observations of primate society, in which the number of individuals in a group is usually not extremely high. There are 20 robots patrolling the field in the simulation. Moreover, although the primate group size changes due to birth and death, the group usually does not split to smaller groups or merge with other groups to form a larger group. A primate group barely dynamically determines its group size based on some trade-offs or environments. This is the reason why in our simulation we set the group size as a fixed value for each group. Our future research will consider for changing group size. We group these robots into either small groups or large groups. If we group them into large groups, 10 robots are in each group, as shown in Fig. 3(a); if we group them into small groups, four robots are in each group, as shown in Fig. 3(b). The communication range is set to 10 units. The sensing/detection range is the same within each group and is set to 10 units. Each robot's successful transmissions influence the successful transmissions of the group. In the simulations, we set each robot's successful transmission possibilities to be 85%, 90%, 95%, and 99%, respectively, and checked how they influence the successful transmissions of the group.

The robots' jobs are to detect and clear the intruders. In this simulation, we set 1–8 intruders for both large and small groups and compare the time slots consumed for the groups to patrol the field and clear the intruders.

The intruders are shown in Fig. 3 as kite-shaped objects wandering in the field. There are six intruders remaining in Fig. 3(a), while there are only three intruders remaining in Fig. 3(b). We assume that it requires 1000 attacks from the robots for an intruder to be cleared. Note that the beating model here is different from the model in Section II. Therefore, if there are four robots in a group, this specific intruder may be cleared after each robot attacks it 250 times. In other words, the larger the group, the less time is required to clear an intruder. A courier (an UAV) flies among all the groups constantly in order to achieve intergroup communication. The courier is shown in Fig. 3 as a compass-shaped object that is flying among all of the groups. We calculate the average time slots per iteration that the courier flies over all of the groups and then compare the courier iteration times. The data obtained by the courier is actually for intergroup communication, so that a group knows whether other groups have encountered attackers. The courier can also be used as a relay node that transmits patrol status of the groups to an outer administrator. Therefore, we place the courier in the simulation for some potential functions.

B. Video Demos of Simulations

Video clips of some of our simulations are available at [25]. Fig. 3 shows the screenshots of the video clips. These video clips help to understand the procedure of the simulations. Two sizes (10 robots per group and four robots per group) are easily distinguished in our video clips.

Our video clips show the simulations of only four cases: 1) large grouping when there are eight intruders, and the successful transmission probability for each robot is 85%; 2) small grouping when there are eight intruders, and the successful transmission probability for each robot is 85%; 3) large grouping when there are eight intruders, and the successful transmission probability for each robot is 99%; and 4) small grouping when there are eight intruders, and the successful transmission probability for each robot is 99%.

The conclusions of the videos are summarized in the next subsection, along with more simulation cases.

C. Simulation Results

For each simulation, we generate graphic simulation results similar to the video clips in Fig. 3. Fig. 4 shows the patrol time (measured by number of time slots) for both large and small grouping. For a given number of intruders, the patrol time fluctuates according to the robots' successful transmission possibilities (p). The communication range and sensing/detection range are set to 10 units. Therefore, the robots in a small group may communicate with each other through a one-hop connection, while there might be multihop connections in a large group. We observe that the lower the robot's successful transmission possibility, the longer the patrol time is. For a given p, the patrol time is different for different numbers of intruders in the field. As illustrated in Fig. 4, when the number of intruders is smaller than four, the patrol times of large and small groupings are similar. When the number of intruders is larger than four, the patrol time of the large grouping is much longer than that of the small grouping. It is not difficult to understand that the number of intruders ultimately causes the disparity after watching our video clips in [25]. If there are not too many intruders, whether we group the robots in large or small groups, each group faces at most two intruders and it takes similar time for the groups to clear the intruders. But if the number of intruders is much larger than the number of

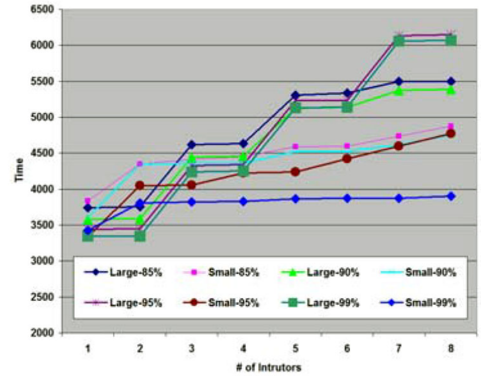


Fig. 4. Robots patrol time.

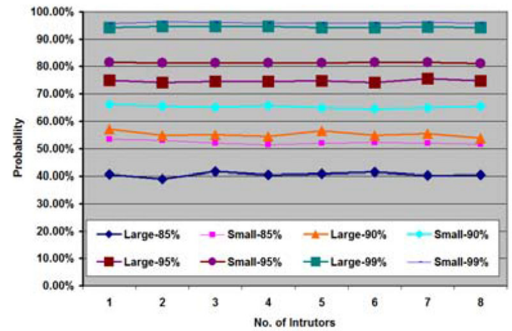


Fig. 5. Group successful transmission possibility.

groups, a large group has to deal with more intruders than a small group. Even though it takes more time for a small group to clear an intruder than a large group, the slow-moving speed caused by the inevitable interference in large groups lengthens its patrol time.

Moreover, although the placement of the intruders affects the patrol time, the curves in Fig. 4 are nondecreasing due to the increasing the number of intruders. The group that spends the longest time on patrolling determines the patrol time of all the robots in the entire field. For example, in the large grouping in which there are only two groups as shown in Fig. 4, the patrol time is similar for five and six intruders. This is because one group may averagely encounter two intruders while the other group may averagely encounter three, if there are totally five intruders. Therefore, the patrol time is determined by the group which encounters three robots.

A small group of robots is able to form a one-hop wireless network, but a large group of robots may not. Moreover, the movement of robots changes the network topology, and thus the successful transmission probability is affected. Therefore, we demonstrate the successful transmission probabilities for both small and large grouping in the simulations. Fig. 5 shows that, for a fixed successful transmission probability for each robot, the successful transmission probability of the small grouping is higher than that of the large grouping. The number of intruders does not greatly influence the successful transmission probability of the group because an intruder is not a node of the wireless sensor network. The

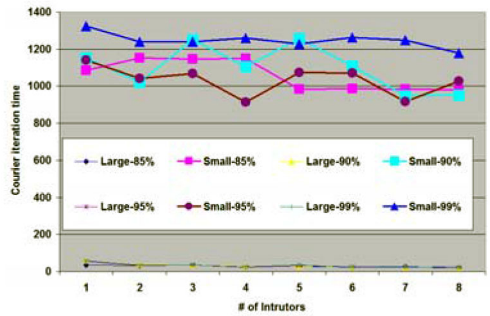


Fig. 6. Courier iteration time.

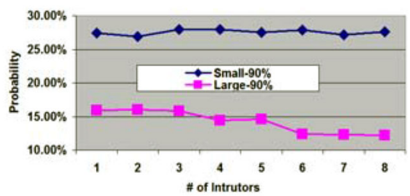


Fig. 7. Probability when # of robots is large.

results would be different, however, if the intruder is able to make channel attacks or other kinds of attacks to damage the wireless network. We will research this case in our future work.

We recorded the time that the courier flies over all of the groups for a number of iterations and calculated the average iteration. Fig. 6 shows that the courier iteration time of the small grouping is much longer than that of the large grouping. It is not difficult to understand this disparity after watching our video clips [25]. The courier needs to fly over all of the groups. In the large grouping, the number of groups is small, while in the small grouping, the number of groups is large. This difference leads to the different routes in the two kinds of grouping, and the route in small grouping has more curves than that of the large grouping.

Although a primate group usually consists a small amount of members, to obtain a more reliable result we set the group size to large values in the following simulations. Fig. 7 shows the successful transmission probability of the group for both large and small grouping when there are 2000 robots patrolling the field in the simulation. Same as the results for a small number of robots, the successful transmission probability of the small grouping is still larger than that of the large grouping when the total number of robots is very large. The number of intruders does not greatly influence the successful transmission probability of the group. We obtain similar results of patrol time and courier iteration time for a large number of robots as well, but the figures are omitted due to space limitation.

In summary, the patrol time of the small grouping is shorter than that of the large grouping if the number of intruders is not small; the successful transmission probability of the small grouping is higher than that of the large grouping; the courier iteration time of the large grouping is shorter than that of the small grouping.

IV. Theoretical Studies

In this section, we provide some theoretical studies under some further assumptions different than in Sections II and III. As stated before, although the simulation and the theoretical study are based on same problem definition, background, and motivation, we expect the theoretical study in this section to present us some other understandings about robot grouping.

A. Assumptions

We use the method of dividing the field into cells and regard the motion of a robot as moving from one cell to its adjacent cell. The field can be divided into different small mutually exclusive cells. The sensing range of a robot can cover each cell, and the area of the cell is roughly equal to the sensing range of a robot. In practice, the area of the cells on the perimeter of the field may be smaller than the sensing range of a robot and can be rounded to be the same as the sensing range of a robot.

The following assumptions apply in this section only, but most of the assumptions in this section are the same as those in our simulations in Section III. We assume that the coordination of the robots within the same group takes a very short time compared to the moving time and the time required for a robot to clear an intruder; and thus the time of coordination can be regarded as zero. This assumption accords to the fact that nowadays robots can be implemented in platforms, which feature high-speed computation and communication. We denote the time when the robots begin to detect intrusions as zero. In other words, at time zero the field has been divided into subfields and one group of robots has been assign to each subfield. There are some other assumptions as shown in Table I.

Remark 1: From Assumptions 3 and 7, it will take a robot group of N_0 robots a time of $(\lceil C_0/N_0 \rceil - 1)t_0$ to cover a subfield that contains C_0 cells. Note that at time zero, N_0 cells have been covered by the robots, and thus the robots only need to cover the remaining cells, the amount of which is $\max(C_0 - N_0, 0)$.

Remark 2: For Assumption 7, there are scenarios in which the assumption applies. Fig. 8 illustrates one of those scenarios that are applicable in practice. In this scenario, the movement of a group of three robots has five stages. In the first stage, the robots advance in line in the left-to-right direction at an average speed V move-by-move, with each move to an adjacent cell, until they reach the boundary of the field in their moving direction; the first stage counts eight moves, and ultimately those moves cost them a time of $8t_0$. Furthermore, in the second stage each of them advances in the top-to-bottom direction at a higher average speed, $V + a$, to a cell three cells down in the new direction; this stage counts one move and takes the robot time t_0. Then, in the third stage the robots advance in a way similar to the first stage except that they advance in the right-to-left direction; this stage takes them a time of $8t_0$. In the fourth stage, since there are only two line of cells left and there are three robots, the robots should not and cannot be kept in a line; then the robots advance in a counter-clockwise direction at a speed $V + b$ until each of them reaches the cells three cells away (note that adjacent

TABLE I
OTHER ASSUMPTIONS FOR THIS SECTION

1	all robots have the same sensing range.
2	For every intrusion/target, after it invades the field, it keeps still. This scenario can be found in the case of target search. The position where an intruder invaded is random and evenly distributed in the cells. In other words, if there are C cells, the probability that an intruder occurs in the i^{th} cell is $1/C$.
3	It takes a time of t_0 for a robot to move from one cell to one of its adjacent cells.
4	When an intruder is detected by a group of robots (a single robot can be regarded as a robot group of size one), the time required to clear the intruder is t_1/m, where m is size of the group.
5	Assume that once a group of robots finds an intruder, none of the robots will move to the new cells (the cells not yet covered) until the intruder is cleared.
6	From the perspective of application, it is reasonable to assume that the total number (denoted as N) of robots is significantly smaller than the number (denoted as C) of cells in the field. Furthermore, in each subfield, the total number of robots is smaller than the number of cells.
7	Assume that in every subfield (the whole field can also be regarded as a subfield) the robots in the same group move in the same time (keep the same pace). In every movement of each of the robots (the robots may be in different groups) in the subfield, every robot will move to a new cell that is not previously covered if possible (the exception is when the new cells left are less than the robots in the group), and each pair of robots will not move to the same new cell. In practice, there are many scenarios that apply this assumption. For example, when the robots in a group move in a line, as shown as in Fig. 8, this assumption applies. We also assume that each move of the robots take a constant time t_0. This assumption implies that the robots can move in varying speed. In Fig. 8 for example, from time t to $t+8t_0$, in each move of the robots, to cover a new cell, each robot only needs to move to an adjacent cell of their current place, while when the robots make a move beginning at $t+8t_0$, for each of the robots to cover the new cells shown as their locations in $t+9t_0$ in Fig. 8, each robot needs to move to a location three cells always from its current location and thus needs to move in a speed two times faster.
8	In the case that the field is divided into subfields (the field with no division can be viewed as divided into one subfield), only one group of robots can be assigned to each subfield; a robot group moves to detect intrusions in the subfield to which it is assigned and cannot move out of that subfield; any intrusion occurring in a subfield can and only can be cleared by the robot group detecting in that subfield.

cells are one cell away from each other) from the cell it was in before the move; this stage counts one move and costs the robot a time of t_0. In the last stage, the robots advance in a clockwise-counterclockwise-clockwise (CCC) direction with a speed of $V + c$, move-by-move. In each move, each of the robots advances in the CCC direction until it reaches the cell that is three cells away from the cell it was in before the move, and the move takes the robots a time of t_0. In the case that, just before the last move, there are only n ($n = 1$ or 2) cells left, each of the robots advances until it reaches the cell n cells away its original position; thus the last stage takes the robots a time of $\lceil K/3 \rceil t_0$, where K is the number of cells remaining before the last stage.

B. Notations

Let $X_1{:}X_K$ denote $X_1, X_2 \ldots, X_K$. We first consider the partition of the field (denoted as F), where partition refers to dividing the field into subfields. Every possible partition can be denoted by a tuple $(F_1{:}F_l, l)$, where l is the number of subfields and $l = 1, 2, \ldots$, F_i is one of the subfields, $F_i \neq \varnothing$, $F_1 \bigcup F_2 \bigcup \ldots \bigcup F_l = F$, $F_i \bigcap F_j = \varnothing$, and the number of

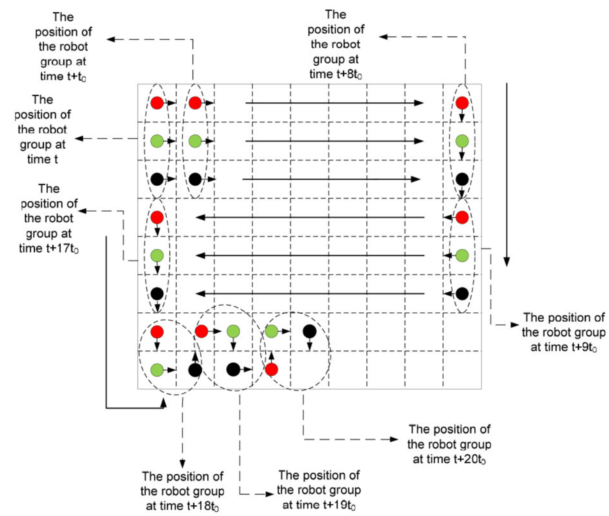


Fig. 8. Scenario of robots moving according to Assumption 7.

cells C_i in F_i satisfies $C_i \geq C_{i+1}$. From Assumption 2, it is reasonable to consider subfields containing the same number of cells to be identical. Then partitions having the same number l of subfields and the same decreasing permutation $(C_1{:}C_l)$ of the amounts of cells in each subfield can be considered to be identical. Then a partition can be uniquely denoted by the tuple $(l, C_1{:}C_l)$, where l is number of subfields, $(C_1{:}C_l)$ is the decreasing permutation of the amounts of cells in the subfields, and $\sum_{i=1}^{l} C_i = C$.

Given a partition of the field, groups of robots need to be assigned to the subfields, and we use the term robot assignment to describe this action. Since in any subfield of any partition there is only one group of robots, considering a partition $(l, C_1{:}C_l)$, we can denote the robot assignment as $(G_1, G_2, \ldots, G_l)$, where G_i denotes the group of robots in the subfield containing C_i cells. Since we consider the subfields having the same amounts of cells to be identical, we can use the permutation $(N_1{:}N_l)$ of the robot amounts of the groups to uniquely denote the groups of robots in the subfields; here N_i is the amount of robots in the robot group that is in the subfield containing C_i cells. To make the tuple $(N_1{:}N_l)$ unique for a given partition, we use the following rule to order the amounts of robots in different robot groups that are in subfields having the same amount of cells.

If $C_i = C_{i+1}$ and $N_i \geq N_{i+1}$, where $\sum_{i=1}^{l} N_i = N$. From above, given a partition $(l, C_1{:}C_l)$, a tuple $(N_1{:}N_l)$ that is defined above may be used to uniquely represent a robot assignment for the partition. In the remainder of this section, we will see that the combination $(l, C_1{:}C_l, N_1{:}N_l)$ of the partition $(l, C_1{:}C_l)$ of the field and the robot assignment $(N_1{:}N_l)$ for the partition can affect the time required for all the N robots to finish their task, which is either a patrolling task or a searching task.

Finally, we denote the set of all possible combinations of partition and robot assignment as Ω, where $\Omega = \{x | x = (l, C_1{:}C_l, N_1{:}N_l)\}$.

C. Two Scenarios

We consider two different scenarios, the searching scenario and the patrolling scenario, according to two different kinds of robot tasks. In the searching scenario, the robots try to find targets as if they were carrying out a search task; the robots know the number of targets but not their positions, and the robots will not detect after all of the targets are detected and cleared, even if there are still cells that have not been covered. In the patrolling scenario, the robots carry a patrolling task; the robots have no information about the targets, such as their numbers of positions, and the robots will cover all of the cells in the field to clear all of the targets. In each of the scenarios, the time required for the robots in the field to finish the task is a random variable whose probability distribution depends on the number of intrusions in the field and the combination of the partition and robot assignment.

Let $T_{i,j}$ denote the time required for N robots to finish their task in scenario i ($i = 1$ or 2) when there are j intrusions in the field; here $i = 1$ signifies a searching scenario, and $i = 2$ indicates that the robots are carrying out a patrolling task. In the following section, we give the deduction of the expression of the expected value $E(T_{i,1})$ of the time required for all the robots in the field to carry out the task in scenario i given that there is one intrusion in the field. We also study the optimal combination of partition and robot assignment that leads to the least $E(T_{i,1})$.

D. Case of One Intruder

In this subsection, we study the expression of $E(T_{i,1})$ and the condition under which $E(T_{i,1})$ becomes least.

1) Scenario 1: Carrying Out a Searching Task: When we gave the definition of $T_{i,j}$ in the previous part, we illustrated that $E(T_{i,j})$ is the function of $(l, C_1{:}C_l, N_1{:}N_l)$. For $T_{1,1}$, from Assumption 9, $T_{1,1} = T_d + T_c$, where T_d and T_c are the times required for all the robots to detect/find the intrusion and the robots to clear it, respectively. T_d and T_c are also functions of $(l, C_1{:}C_l, N_1{:}N_l)$. We can thus obtain that

$$E(T_{1,1})(l, C_1{:}C_l, N_1{:}N_l) = E(T_d)(l, C_1{:}C_l, N_1{:}N_l) + E(T_c)(l, C_1{:}C_l, N_1{:}N_l).$$

Theorem 1 shows that $(1, C, N) = \arg\min_x \{E(T_{1,1})(x) | x \in \Omega\}$; that is to say, in the searching scenario when there is only one intrusion in the field, keeping the field as a whole and assigning all the available robots is the best strategy in terms of the time required for all the robots to carry out the task.

The proofs of this paper can be found in Appendix. We give the expression of the shortest time $\min_x \{E(T_{1,1})(x) | x \in \Omega\}$ required to carry out the task.

Theorem 1: In Scenario 1, $E(T_{1,1})$ is optimal (reaches the least) when $(l, C_1{:}C_l, N_1{:}N_l) = (1, C, N)$.

Now let us give the expression of $\min_x \{E(T_{1,1})(x) | x \in \Omega\}$. From the proof of Theorem 1, we know that

$$E(T_{1,1})(1, C, N) = E(T_d)(1, C, N) + E(T_c)(1, C, N)$$
$$= \left(\frac{N}{C}\sum_{i=0}^{\lceil C/N\rceil-2} it_0\right) + \left(1 - \left(\left\lceil\frac{C}{N}\right\rceil - 1\right)\frac{N}{C}\right)\left(\left\lceil\frac{C}{N}\right\rceil - 1\right)t_0 + \frac{t_1}{N}.$$

From Theorem 1, we know that

$$\min_x \{E(T_{1,1})(x) | x \in \Omega\} = E(T_{1,1})(1, C, N)$$
$$= \left(\frac{N}{C}\sum_{i=0}^{\lceil C/N\rceil-2} it_0\right) + \left(1 - \left(\left\lceil\frac{\mathrm{C}}{\mathrm{N}}\right\rceil - 1\right)\frac{N}{C}\right)\left(\left\lceil\frac{\mathrm{C}}{\mathrm{N}}\right\rceil - 1\right)t_0 + \frac{t_1}{N}.$$

2) Scenario 2: Carrying Out a Patrolling Task: In the patrolling scenario, the robots have no information about the targets before they begin to detect. Thus, given $(l, C_1{:}C_l, N_1{:}N_l)$, $T_{2,1}$ can be expressed as $T_{2,1} = \max_i\{(\left\lceil\frac{C_i}{N_i}\right\rceil - 1)t_0 + I(\Phi_i)\, t_1/N_i | i = 1, .., l\}$, where Φ_i is the event that the intrusion occurs in subfield i and $I(\Phi_i)$ is 1 if Φ_i is true, and 0 if otherwise. Then, we can prove that the probability distribution of $T_{2,1}$ is

$$P\left\{\begin{array}{l} T_{2,1} = \\ \max\left(\begin{array}{l}\left(\left\lceil\frac{C_1}{N_1}\right\rceil - 1\right)t_0, \ldots, \left(\left\lceil\frac{C_i}{N_i}\right\rceil - 1\right)t_0 + t_1/N_i, \\ \ldots, \left(\left\lceil\frac{C_l}{N_l}\right\rceil - 1\right)t_0\end{array}\right)\end{array}\right\}$$
$$= \frac{C_i}{C}. \tag{1}$$

The distribution above implies that given a combination $(l, C_1{:}C_l, N_1{:}N_l)$, the target has a probability distribution in the position where it occurs—it has a probability of C_i/C of occurring in subfield i which contains C_i cells; the time required to finish a patrolling task is the maximum time among those times, each of which is the time required for the group of robots in a subfield to cover all the cells in the subfield and to clear the target if there is any.

In the following text, we study the conditions under which $E(T_{2,1})$ becomes smaller. As Theorem 2 indicates, when C is a multiple of N, $(1, C, N) = \arg\min_x \{E(T_{2,1})(x) | x \in \Omega\}$; that is to say, keeping the field as a whole and assigning all the available robots is the best strategy in terms of the time required for all the robots to carry out the task. At the end of this section, we give the expression of $\min_x \{E(T_{2,1})(x) | x \in \Omega\}$. For the proof of Theorem 2, we need to first deduce Lemma 1 and Lemma 2.

Lemma 1: Suppose that N_0 robots carries out a patrolling task in a field that contains C_0 cells and that there is only one target in the field. For any two-subfield partition of the field $(2, C_0 - C_1, C_1)$, where $C_1 < C_0$, and any combination of the amounts of the robot groups is $(N_0 - N_1, N_1)$, the weighted time $\frac{(C_0-C_1)}{C_0}\frac{(C_0-C_1)}{N_0-N_1}t_0 + \frac{C_1}{C_0}\frac{C_1}{N_1}t_0$ satisfies $\frac{(C_0-C_1)}{C_0}\frac{(C_0-C_1)}{N_0-N_1}t_0 + \frac{C_1}{C_0}\frac{C_1}{N_1}t_0 \geq (\frac{C_0}{N_0})t_0$. Further, $\frac{(C_0-C_1)}{C_0}\left(\left\lceil\frac{(C_0-C_1)}{N_0-N_1}\right\rceil - 1\right)t_0 + \frac{C_1}{C_0}\left(\left\lceil\frac{C_1}{N_1}\right\rceil - 1\right)t_0 \geq \left(\frac{C_0}{N_0} - 1\right)t_0$.

Lemma 2: Suppose that N_0 robots carries out a patrolling task in a field that contains C_0 cells and that there is only one target in the field. For any partition of the field $\left(l, C_1{:}C_{l-1}, C_0 - \sum_{i=1}^{l-1} C_i\right)$ and any combination of the amounts of the robot groups is $\left(N_1{:}N_{l-1}, N_0 - \sum_{i=1}^{l-1} N_i\right)$, the weighted time $\frac{\left(C_0-\sum_{i=1}^{l-1} C_i\right)}{C_0}\left(\frac{(C_0-\sum_{i=1}^{l-1} C_i)}{N_0-\sum_{i=1}^{l-1} N_i}\right)t_0 + \sum_{i=1}^{l-1}\frac{C_i}{C_0}\left(\frac{C_i}{N_i}\right)t_0$ of the two groups of robots to cover their entire field satisfies: $\frac{\left(C_0-\sum_{i=1}^{l-1}C_i\right)}{C_0}\left(\frac{\left(C_0-\sum_{i=1}^{l-1}C_i\right)}{N_0-\sum_{i=1}^{l-1}N_i}\right)t_0 + \sum_{i=1}^{l-1}\frac{C_i}{C_0}\left(\frac{C_i}{N_i}\right)t_0 \geq \frac{C_0}{N_0}t_0$. Further

$$\frac{(C_0-\sum_{i=1}^{l-1}C_i)}{C_0}\left(\left\lceil\frac{(C_0-\sum_{i=1}^{l-1}C_i)}{N_0-\sum_{i=1}^{l-1}N_i}\right\rceil-1\right)t_0 + \sum_{i=1}^{l-1}\frac{C_i}{C_0}\left(\left\lceil\frac{C_i}{N_i}\right\rceil-1\right)t_0 \geq \left(\frac{C_0}{N_0}-1\right)t_0.$$

Theorem 2: When C is a multiple of N, $E(T_{2,1})$ is optimal (reaches the least) when $(l, C_1{:}C_l, N_1{:}N_l) = (1, C, N)$.

Remark 3: If C is not a multiple of N, $(1, C, N)$ is not necessarily the optimal combination to minimize $E(T_{2,1})$. In this case, the optimal combination to minimize $E(T_{2,1})$ depends on t_0, t_1, N, and C.

From Theorem 2, when C is a multiple of N, $\min_x\{E(T_{2,1})(x)|x \in \Omega\} = E(T_{2,1})(1, C, N) = (\frac{C}{N}-1)t_0 + t_1/N$.

V. Related Work

Researchers have studied the target/intrusion detection problem using moving sensors. In [26] and [27], the evasion pursuing problem was studied in a curved and polygonal environment, respectively. In each work, several solutions are presented by the authors. The work in [28] reveals that in a polygonal environment, using moving sensors with omnidirectional vision can insure that intrusions are caught in polynomial (quadratic) time. In [29], the authors consider the environment to be numbers of cells, the area of each of which accords to a moving sensor's sensing range; a two-step detecting method is used that includes a step of obtaining an estimated track of each target and a step of pursuing the target along the track. In [30] and [31], a robot team that carries out target detection tasks is considered and two game-theory based methods, one in each of the papers, are applied to determine the optimal next-move destination for each of the robots to solve the coordination problem of the robot team.

In all of the references in the last paragraph, the authors considered a robot team and tried to maximize the payoff of the team (the payoff can be the time required to find targets or the probability of finding a target) by optimizing the moving patterns of the robots.

Our method in this paper is different from the methods mentioned in that we considered dividing the environment into subfields and applied a subset of all of the robots to each of the subfields. The reason for this consideration is that we noticed a tradeoff between keeping robots in large groups and keeping them in smaller groups. The tradeoff is that a larger group can clear an intrusion in a shorter time than a smaller group, but a bigger group may take more coordination time (time required for communication between group members) and that smaller groups can work in a parallel way that may be more sufficient when there are numbers of intrusions. One of our aims in this paper is to determine the best strategy to divide the environment and to assign the robots. A preliminary result of this paper was presented in the conference [34].

VI. Conclusion

We utilize studies of grouping in primate species to inspire our communication and networking approaches, as well as to examine the patrol time, successful transmission probability, and courier iteration time of both large and small groups. We provide theoretical studies to model the large and small grouping. Small grouping is best for patrol time and successful transmission probability, while large grouping is best for the courier iteration time.

For the future work, we will consider the intruder that is able to make channel attack or other kinds of attacks to damage the wireless sensor network. We have not given a tradeoff between the benefit and loss of a grouping in this paper. In our future work, the benefit and cost of a grouping will be calculated; then the precise size of the best group will be able to be determined.

The intruders do not behave as they have intelligence that they can make smart moves (e.g., avoid robot patrols, group together, move to positions that have been patrolled, destroy the robots, etc.) in this paper. A more realistic opponent model that also affects which group size performs better would be proposed in the future work.

Appendix

Proof of Theorem 1: From Assumption 7, we know that T_d is a random variable and that the distribution of T_d depends on the combination $(l, C_1{:}C_l, N_1{:}N_l)$ of the partition $(l, C_1{:}C_l)$ and robot amounts $(N_1{:}N_l)$ of the groups in the subfields and has nothing to do with the motion of each group in each subfield. This fact implies that $E(T_d)$ and $E(T_c)$ are functions of $(l, C_1{:}C_l)$ and $(N_1{:}N_l)$.

Let $E(T_d)(l, C_1{:}C_l, N_1{:}N_l)$ and $E(T_c)(l, C_1{:}C_l, N_1{:}N_l)$ denote the expected time required to detect the intrusion and the expected time required to clear the intrusion in the case of partition $(l, C_1{:}C_l)$ and robot amounts $(N_1{:}N_l)$, respectively. In the following, we will prove that, for any possible $(l, C_1{:}C_l, N_1{:}N_l)$:

$$E(T_d)(1, C, N) \leq E(T_d)(l, C_1{:}C_l, N_1{:}N_l). \tag{A1}$$

We know that, for the combination $(1, C, N)$, there is one subfield containing C cells where a group consisting of all N robots detects intrusions. From Assumption 7, for the case of the combination of $(1, C, N)$, the distribution of T_d is

$$P(T_d = it_0) = \begin{cases} N/C \text{ if } 0 \leq i \leq \lceil C/N \rceil - 2; \\ 1 - \ (\lceil C/N \rceil - 1)N/C \text{ if } i = \lceil C/N \rceil - 1; \\ 0 \text{ if } i > \lceil C/N \rceil - 1. \end{cases} \tag{A2}$$

Also from Assumption 7, in the case of the combinations $(l, C_1{:}C_l, N_1{:}N_l)$ other than $(1, C, N)$, the distribution of T_d satisfies

$$\begin{cases} P(T_d = it_0) = p_i(l, C_1{:}C_l, N_1{:}N_l) \\ \leq N/C, \ \text{ if } 0 \leq i \leq \lceil C/N \rceil - 2; \\ P(T_d \geq (\lceil C/N \rceil - 1)t_0) = 1 - P(T_d \leq (\lceil C/N \rceil - 2)t_0) \\ \geq 1 - (\lceil C/N \rceil - 1)N/C. \end{cases} \tag{A3}$$

From (A3), for any combination $(l, C_1{:}C_l, N_1{:}N_l)$ other than $(1, C, N)$, we have

$$E(T_d)(l, C_1{:}C_l, N_1{:}N_l) = \sum_{i=0}^{+\infty} it_0 p(t_d = it_0)$$
$$= \sum_{i=0}^{\lceil C/N\rceil-2} it_0 p_i + \sum_{i=\lceil C/N\rceil-1}^{+\infty} it_0.P(t_d = it_0)$$
$$\geq \sum_{i=0}^{\lceil C/N\rceil-2} it_0 p_i + (\lceil C/N\rceil - 1)t_0 \left(1 - \sum_{i=0}^{\lceil C/N\rceil-2} p_i\right) \quad \text{(A4)}$$

where $p_i = p_i(l, C_1{:}C_l, N_1{:}N_l)$.

Then from (A2) we have

$$\begin{aligned}
&E(T_d)(1, C, N)\\
&= \sum_{i=0}^{\lceil C/N\rceil-2} (N/C)it_0 + (\lceil C/N\rceil - 1)t_0(1 - (\lceil C/N\rceil - 1)N/C)\\
&= \sum_{i=0}^{\lceil C/N\rceil-2} it_0 p_i + \sum_{i=0}^{\lceil C/N\rceil-2} it_0(N/C - p_i)\\
&\quad + (\lceil C/N\rceil - 1)t_0(1 - (\lceil C/N\rceil - 1)N/C)\\
&\leq \sum_{i=0}^{\lceil C/N\rceil-2} it_0 p_i + (\lceil C/N\rceil - 1)t_0 \sum_{i=0}^{\lceil C/N\rceil-2} (N/C - p_i)\\
&\quad + (\lceil C/N\rceil - 1)t_0(1 - (\lceil C/N\rceil - 1)N/C)\\
&= \sum_{i=0}^{\lceil C/N\rceil-2} it_0 p_i + (\lceil C/N\rceil - 1)t_0 \left(1 - \sum_{i=0}^{\lceil C/N\rceil-2} p_i\right) \qquad \text{(A5)}
\end{aligned}$$

where $p_i = p_i(l, C_1{:}C_l, N_1{:}N_l)$.

From (A4) and (A5), we can easily obtain (A1). From Assumption 5, we can easily obtain that for any combination $(l, C_1{:}C_l, N_1{:}N_l)$ other than $(1, C, N)$

$$E(T_c)(l, C_1{:}C_l, N_1{:}N_l) > E(T_c)(1, C, N). \qquad \text{(A6)}$$

From $E(T_{1,1}) = E(T_c) + E(T_d)$, (A1), and (A6), for any combination $(l, C_1{:}C_l, N_1{:}N_l)$ other than $(1, C, N)$, we have

$$E(T_{1,1})(l, C_1{:}C_l, N_1{:}N_l) > E(T_{1,1})(1, C, N).\#$$

Proof of Lemma 1:

$$\begin{aligned}
&\tfrac{(C_0-C_1)}{C_0}\left(\left\lceil \tfrac{(C_0-C_1)}{N_0-N_1}\right\rceil - 1\right)t_0 + \tfrac{C_1}{C_0}\left(\left\lceil \tfrac{C_1}{N_1}\right\rceil - 1\right)t_0\\
&= \left\{\tfrac{(C_0-C_1)}{C_0}\left(\left\lceil \tfrac{(C_0-C_1)}{N_0-N_1}\right\rceil\right) + \tfrac{C_1}{C_0}\left(\left\lceil \tfrac{C_1}{N_1}\right\rceil\right)\right\}t_0 - t_0
\end{aligned} \qquad \text{(A7)}$$

$$\begin{aligned}
&\tfrac{(C_0-C_1)}{C_0}\left\lceil \tfrac{(C_0-C_1)}{N_0-N_1}\right\rceil + \tfrac{C_1}{C_0}\left\lceil \tfrac{C_1}{N_1}\right\rceil \geq\\
&\tfrac{(C_0-C_1)}{C_0}\tfrac{(C_0-C_1)}{N_0-N_1} + \tfrac{C_1}{C_0}\tfrac{C_1}{N_1} = g(C_1)
\end{aligned} \qquad \text{(A8)}$$

$$\begin{aligned}
g(C_1) &= \frac{(C_0-C_1)}{C_0}\frac{(C_0-C_1)}{N_0-N_1} + \frac{C_1}{C_0}\frac{C_1}{N_1}\\
&= \frac{(C_0-C_1)(C_0-C_1)N_1 + C_1^2(N_0-N_1)}{C_0(N_0-N_1)N_1}\\
&= \frac{N_0C_1^2 - 2C_0N_1C_1 + C_0^2N_1}{C_0(N_0-N_1)N_1}.
\end{aligned}$$

Since $g(C_1)$ is a quadratic function of C_1 and $\frac{N_0}{C_0(N_0-N_1)N_1} > 0$, we know that

$$g(C_1) \geq g(C_0N_1/N_0) = C_0/N_0. \qquad \text{(A9)}$$

From (A7)–(A9), Lemma 1 is proved. #

Proof of Lemma 2: We can obtain that

$$\begin{aligned}
&\frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{C_0}\left(\left\lceil \frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{N_0 - \sum_{i=1}^{l-1} N_i}\right\rceil - 1\right)t_0\\
&\quad + \sum_{i=1}^{l-1} \frac{C_i}{C_0}\left(\left\lceil \frac{C_i}{N_i}\right\rceil - 1\right)t_0 \qquad \text{(A10)}\\
&= \left\{\frac{(C_0 - \sum_{i=1}^{l-1} C_i)}{C_0}\left(\left\lceil \frac{(C_0 - \sum_{i=1}^{l-1} C_i)}{N_0 - \sum_{i=1}^{l-1} N_i}\right\rceil\right)\right.\\
&\quad \left. + \sum_{i=1}^{l-1} \frac{C_i}{C_0}\left\lceil \frac{C_i}{N_i}\right\rceil\right\}t_0 - t_0
\end{aligned}$$

$$\begin{aligned}
&\frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{C_0}\left\lceil \frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{N_0 - \sum_{i=1}^{l-1} N_i}\right\rceil + \sum_{i=1}^{l-1} \frac{C_i}{C_0}\left\lceil \frac{C_i}{N_i}\right\rceil\\
&\geq \frac{(C_0 - \sum_{i=1}^{l-1} C_i)}{C_0}\frac{(C_0 - \sum_{i=1}^{l-1} C_i)}{N_0 - \sum_{i=1}^{l-1} N_i} + \sum_{i=1}^{l-1} \frac{C_i}{C_0}\frac{C_i}{N_i} = f(l). \qquad \text{(A11)}
\end{aligned}$$

We use the method of induction to prove it. From Lemma 1, we know that Lemma 2 is true when $k = 2$. Now suppose that, when $k = l - 1$, Lemma 2 is true. From this, we can know that

$$\sum_{i=1}^{l-1} \frac{C_i}{C_0}\frac{C_i}{N_i} = \frac{\sum_{i=1}^{l-1} C_i}{C_0}\left(\sum_{i=1}^{l-1} \frac{C_i}{\sum_{i=1}^{l-1} C_i}\frac{C_i}{N_i}\right) \geq \frac{\sum_{i=1}^{l-1} C_i}{C_0}\left(\frac{\sum_{i=1}^{l-1} C_i}{\sum_{i=1}^{l-1} N_i}\right). \qquad \text{(A12)}$$

From (A11) and (A12), we know that

$$f(l) \geq \frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{C_0}\frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{N_0 - \sum_{i=1}^{l-1} N_i} + \frac{\sum_{i=1}^{l-1} C_i}{C_0}\frac{\sum_{i=1}^{l-1} C_i}{\sum_{i=1}^{l-1} N_i}. \qquad \text{(A13)}$$

From (A13) and Lemma 1, we can obtain that

$$f(l) \geq \frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{C_0}\frac{\left(C_0 - \sum_{i=1}^{l-1} C_i\right)}{N_0 - \sum_{i=1}^{l-1} N_i} + \frac{\sum_{i=1}^{l-1} C_i}{C_0}\frac{\sum_{i=1}^{l-1} C_i}{\sum_{i=1}^{l-1} N_i} \geq \frac{C_0}{N_0}. \qquad \text{(A14)}$$

From (A10), (A11), and (A14), we know that, when $k = l$, Lemma 2 is true. Then Lemma 2 is proved.

Proof of Theorem 2: For any combination $(l, C_1{:}C_l, N_1{:}N_l)$ of the partition of the field and the amounts of the robots in the groups of the subfields other than $(1, C, N)$, from (1), we know that

$$\begin{aligned} &E(T_{2,1})(l, C_1{:}C_l, N_1{:}N_l)\\ &= \sum_{i=1}^{l}\left\{\frac{C_i}{N_1}\max\left(\left(\left\lceil\frac{C_1}{N_1}\right\rceil - 1\right)t_0, \ldots, \left(\left\lceil\frac{C_i}{N_i}\right\rceil - 1\right)t_0\right.\right.\\ &\quad\left.\left. + t_1/N_i, \ldots, \left(\left\lceil\frac{C_l}{N_l}\right\rceil - 1\right)t_0\right)\right\}. \qquad \text{(A15)} \end{aligned}$$

From (1), we can obtain

$$\begin{aligned} &E(T_{2,1})(l, C_1{:}C_l, N_1{:}N_l)\\ &\geq \sum_{i=1}^{l}\left\{\frac{C_i}{C}\left(\left(\left\lceil\frac{C_i}{N_i}\right\rceil - 1\right)t_0 + t_1/N_i\right)\right\}\\ &= \sum_{i=1}^{l}\left\{\frac{C_i}{C}\left(\left\lceil\frac{C_i}{N_i}\right\rceil - 1\right)\right\}t_0 + \sum_{i=1}^{l}\frac{C_i}{C}t_1/N_1\\ &> \sum_{i=1}^{l}\left\{\frac{C_i}{C}\left(\left\lceil\frac{C_i}{N_i}\right\rceil - 1\right)\right\}t_0 + t_1/N. \qquad \text{(A16)} \end{aligned}$$

From (A16) and Lemma 2, we know that, for any combination $(l, C_1{:}C_l, N_1{:}N_l)$ other than $(1, C, N)$

$$E(T_{2,1})(l, C_1{:}C_l, N_1{:}N_l) > (C/N - 1)t_0 + t_1/N. \qquad \text{(A17)}$$

In fact, when C is a multiple of N, $E(T_{2,1})(1, C, N) = (C/N - 1)t_0 + t_1/N$. From this fact and (A17), we know that keeping all the robots as a group in the field with no partition leads to a minimum of time required to carry out the patrolling task in the case that there is only one target. #

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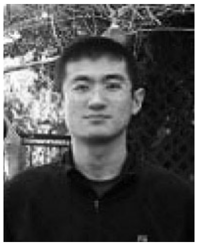

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